Mathematical Methods in Medicine and Biology
Due March 2, 2000

Consider the Hodgkin-Huxley partial differential equations for the giant squid axon.

$$C_m \frac{\partial V_m}{\partial t} + G_{Na} m^3 h (V_m - E_{Na}) + G_K n^4 (V_m - E_K) + G_L (V_m - E_L) = \frac{a}{2\rho} \frac{\partial^2 V_m}{\partial x^2}$$

where

$$\frac{\partial m}{\partial t} = \alpha_m (1 - m) - \beta_m m$$
$$\frac{\partial h}{\partial t} = \alpha_h (1 - h) - \beta_h h$$
$$\frac{\partial n}{\partial t} = \alpha_n (1 - n) - \beta_n n$$

1. Write a program to solve the system using the Split Backward-Euler method described in class. To do this, update m, n, h, and V_m using their old values. Use $dx = 0.1$ cm. Turn in code. Create an action potential and turn in graph.

2. Allow the system to come to equilibrium. Give the resting system, a +50 mV pulse (add 50 mV to the resting value of V_m) to create a propagating action potential. Graph the result by showing the spatial profile at two different times. Be sure to label graph axes (include units). Calculate the speed of the propagating action potential.

Hodgkin-Huxley Units

voltage: mV
current: μA
time: ms
length: cm
unit conductance: $\mu A \cdot mV^{-1} \cdot cm^{-2}$
unit capacitance: $\mu A \cdot ms \cdot mV^{-1} \cdot cm^{-2} = \mu F \cdot cm^{-2}$

Hodgkin-Huxley Parameters

$C_m = 1.0\mu F \cdot cm^{-2}$
$G_{Na} = 120.0\mu A \cdot mV^{-1} \cdot cm^{-2}$
$G_K = 36.0\mu A \cdot mV^{-1} \cdot cm^{-2}$
$G_L = 0.3\mu A \cdot mV^{-1} \cdot cm^{-2}$
$E_{Na} = 45.0mV$
$E_K = -82.0.0mV$
$E_L = -59.0mV$
$a = 0.0238cm$
$\rho\ = 35.4\Omega cm = 0.0354mV \cdot \mu A^{-1} \cdot cm$

Hodgkin-Huxley Functions

$$
\begin{aligned}
\alpha_m(V) &= \frac{V+45}{10(1-e^{\frac{v+45}{-10}})} \\
\beta_m(V) &= 4e^{\frac{V+70}{-18}} \\
\alpha_n(V) &= 0.1\frac{V+60}{10(1-e^{\frac{v+60}{-10}})} \\
\beta_n(V) &= 0.125e^{\frac{V+70}{-80}} \\
\alpha_h(V) &= 0.07e^{\frac{V+70}{-20}} \\
\beta_h(V) &= \frac{1}{(1+e^{\frac{v+40}{-10}})}
\end{aligned}
$$

V in mV and α and β in s^{-1}.